NBV-003-1262003 Seat No. ________

M. Phil (Sem. II) (CBCS) Examination

April / May - 2017

Mathematics : EMT-20011

(Complex Analysis)

(New Course)

Faculty Code : 003

Subject Code : 1262003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Answer all questions.

(2) Each question carries **14** marks.

1 Answer any **seven** questions : **2×7=14**

(i) If $a \in \not\subset, |a| <|$ then prove that $\phi_a(z) = \frac{z-a}{1-\bar{a}z}$ is analytic on $\left\{ z \in \not\subset \Big| |z| < \frac{2}{|a|} \right\}$.

(ii) If $f(z) = \exp(e^z) \forall z \in \not\subset$ then find $M(r), \forall \sim > 0$.

(iii) True or False ? Justify :

Weirstrass factorization of an entire function is unique.

(iv) If f is a non-constant entire function then prove that $M(\sim) \to \infty$ as $r \to \infty$.

(v) Prove that $0, \infty$ are asymptotic values of the exponential function.

(vi) State, without proof, Hadamard's factorization theorem.

(vii) For an entire function f, prove that

$a \in pv(f) \Rightarrow$ "a" is an asymptotic value of f.

(viii) If f is an entire function of finite non-integral order then prove that f has infinitely many zeros.

(ix) Prove that the only analytic functions : $\not\subset_{\infty} \to \not\subset$ are constant functions.

(x) If $w_1, w_2 \in \not\subset$ are linearly independent over $\mathbb{R}$ and $\Pi = \{nw_1 + mw_2 \mid n, m \in z\}$ then prove that $f : \not\subset / \Pi \to S^1 \times S^1$ defined by $f((\alpha w_1 + \beta w_2) + \Pi) = \left(e^{2\pi i \alpha}, e^{2\pi i \beta}\right)$, $\forall \alpha, \beta \in \mathbb{R}$ is one-one.

2 Answer any **two** questions : **2×7=14**

(a) State and prove Jensen's formula.

(b) Define order of an entire function. Find the order of cosz.

(c) Give an example of an entire function with infinite order. Justify.

3 (a) If f is an entire function of order λ then prove that **7**

$$\lambda = \limsup_{r \to \infty} \frac{\log \log m(\sim)}{\log r}.$$

(b) If f is an entire function, $f(0) = 1$ and $n(r)$ is the number of zeros of f in $B(0, r)$ counted according to the multiplicity, $\forall \sim > 0$ then prove that $n(r) \log 2 \le \log M(2r), \forall \sim > 0.$ **7**

OR

3 (c) If $GC \not\subset$ is a region, $f : G \to \not\subset$ is analytic, $f(z) \ne 0, \forall z \in G$ and $h(z) = \log |f(z)|, \forall z \in G$ then prove that $\frac{\partial h}{\partial x} - \frac{i \partial h}{\partial y} = \frac{f^1}{f}$ on G. **7**

(d) Let g be a polynomial of degree $n \ge 1$ then prove that order of $e^{g(z)}$ is n. **7**

4 Answer any **two** questions : **2×7=14**

(a) True or False ? Justify. The exponential function has only finitely many fixed points.

(b) If $f : B(a,\sim) \to \not\subset$ is analytic and $|f^1(z) - f^1(a)| < |f^1(a)|$,

$\forall a \neq z \in (a,\sim)$ then prove that $f : (B(a,\sim) \to \not\subset$ is one-one.

(c) State and prove Little Picard theorem.

5 Answer any **two** questions : **2×7=14**

(a) If $g : B(0,R) \to \not\subset$ is analytic, $g(0) = 0, |g^1(0)| = \mu > 0$

and $|g(z)| \leq M, \forall z \in B(0,R)$ then prove that

$$g(B(o,R)) \supset B\left(0, \frac{R^2 \mu^2}{6M}\right).$$

(b) If $f : \not\subset \to \not\subset$ is entire and injective then prove that f is a polynomial of degree 1.

(c) With usual notation, prove that $I\!P^1(\not\subset)$ is a compact Riemann surface.

(d) Define analytic function between two Riemann surfaces. Prove that every polynomial with complex coefficients is an analytic function : $\not\subset_\infty \to \not\subset_\infty$.
